Plan

1 Om eksamen
2 Gjennomgang: Eksamen MET1180 fra 05/2022

① Om eksamen

Eksamen 5t : 15 delspørsmål
Hjelpemidler: : Kalkulator + formelsamling

Tema: * Matriser og vektorer 1/3
* Integrasjon 1/3
* Funksjoner i to variabler 1/3
+ Emner fra høstsemesteret
– løse likninger/ulikheter (polynomdiv., fullføre kvadrat, faktor., fortegnsskjema)
– geometri (sirkler/ellipser, hyperbler, parabler, rette linjer)
– finansmatematikk
– optimering i en variabel (derivasjon)

Forberedelser: – eksamen fra 23/05/2022
– fasoppgaven fra mars
– oppgavesett 25 – 48
– eldre eksamensoppgaver

2 Eksamen 05/2022

1. a)

$$\left(\begin{array}{cccc|c} 2 & -6 & 4 & 6 & 8 \\ 3 & -12 & 7 & 2 & 7 \\ 1 & -2 & 1 & 10 & 5 \end{array}\right) \to \left(\begin{array}{cccc|c} 1 & -2 & 1 & 10 & 5 \\ 3 & -12 & 7 & 2 & 7 \\ 2 & -6 & 4 & 6 & 8 \end{array}\right) \begin{array}{l} -3 \\ \\ -2 \end{array}$$

$$\to \left(\begin{array}{cccc|c} 1 & -2 & 1 & 10 & 5 \\ 0 & -6 & 4 & -28 & -8 \\ 0 & -2 & 2 & -14 & -2 \end{array}\right) \to \left(\begin{array}{cccc|c} 1 & -2 & 1 & 10 & 5 \\ 0 & -2 & 2 & -14 & -2 \\ 0 & -6 & 4 & -28 & -8 \end{array}\right) -3$$

$$\to \left(\begin{array}{cccc|c} 1 & -2 & 1 & 10 & 5 \\ 0 & -2 & 2 & -14 & -2 \\ 0 & 0 & -2 & 14 & -2 \end{array}\right)$$

trappeform

$$x - 2y + z + 10w = 5$$
$$-2y + 2z - 14w = -2$$
$$-2z + 14w = -2$$

w fri uendelig mange løsn.

$$z = \frac{-2z}{-2} = \frac{-14w-2}{-2} = 7w+1$$

$$\frac{-2y}{-2} = \frac{-2(7w+1) + 14w - 2}{-2} = \frac{-4}{-2}$$

$$y = 2$$

$$x = 2(2) - (7w+1) - 10w + 5 = -17w + 8$$

Løsn:

$$(x, y, z, w) = (-17w + 8, 2, 7w+1, w)$$

der w er fri

b)

$$\left(\begin{array}{cccc|c} 2 & -6 & 4 & 6 & 8 \\ 3 & a & 7 & 2 & 7 \\ 1 & -2 & 1 & 10 & 5 \end{array}\right) -1 \to \left(\begin{array}{cccc|c} 1 & -4 & 3 & -4 & 3 \\ 3 & a & 7 & 2 & 7 \\ 1 & -2 & 1 & 10 & 5 \end{array}\right) \begin{array}{l} -3 \\ -1 \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & -4 & 3 & -4 & 3 \\ 0 & a+12 & -2 & 14 & -2 \\ 0 & 2 & -2 & 14 & 2 \end{array}\right) :2 \quad \rightarrow \quad \left(\begin{array}{cccc|c} 1 & -4 & 3 & -4 & 3 \\ 0 & 1 & -1 & 7 & 1 \\ 0 & a+12 & -2 & 14 & -2 \end{array}\right) \cdot -(a+12)$$

$$\rightarrow \left(\begin{array}{cccc|c} 1 & -4 & 3 & -4 & 3 \\ 0 & 1 & -1 & 7 & 1 \\ 0 & 0 & * & ** & *** \end{array}\right)$$

$* = -2 + a + 12 = a + 10$

$** = 14 - 7(a+12) = 7(2 - a + 12) = -7(a+10)$

$a \neq -10$: uendelig mange løsn.

$a = -10$: ~~uendelig mange løsn.~~ Ingen løsn

Konkl.: System er inkonsistent for $a = -10$

~~Konkl.: Systemet har alltid løsn. (for alle a) ⇒ Ingen verdier av a s.a. systemet blir inkonsistent~~

$*** = -2 - (a+12) = -a - 14 = -(a+14)$

2. a) $\displaystyle\int_0^1 6\sqrt{x} - 11x\sqrt[5]{x}\,dx = \int_0^1 6x^{1/2} - 11x^{6/5}\,dx$

$\displaystyle = \left[\cancel{6}\,\frac{2}{\cancel{3}}\cdot x^{3/2} - \cancel{11}\cdot\frac{5}{\cancel{11}}x^{11/5}\right]_0^1 = (4-5) - 0 = \underline{\underline{-1}}$

b) $\displaystyle\int \frac{21-x}{9-x^2}\,dx = \int \frac{4}{3+x} + \frac{3}{3-x}\,dx$

$9 - x^2 = (3+x)(3-x)$

$\displaystyle\frac{21-x}{(3+x)(3-x)} = \frac{A}{3+x} + \frac{B}{3-x} \quad \Big| \cdot (3+x)(3-x)$

$$\boxed{\begin{aligned} 21 - x &= A(3-x) + B(3+x) \\ 21 - x &= (3A+3B) + (B-A)x \end{aligned}}$$

$3A + 3B = 21$
$B - A = -1$

$A + B = 7$
$A - B = 1$

$2A = 8$, $\underline{A = 4}$, $\underline{B = 3}$

$$= \underline{\underline{4 \cdot \ln|3+x| - 3\ln|3-x| + C}}$$

c) $\displaystyle\int \frac{1}{1-\sqrt{x}}\,dx = \int \frac{1}{u}(-2\sqrt{x})\,du$

$$\boxed{\begin{aligned} u &= 1-\sqrt{x} \\ du &= -\frac{1}{2\sqrt{x}}\,dx \end{aligned}} \Rightarrow \sqrt{x} = 1-u$$

$$\Rightarrow dx = -2\sqrt{x}\,du$$

$$= \int \frac{1}{u}\cdot(-2)\cdot(1-u)\,du = \int \frac{2u-2}{u}\,du = \int 2 - \frac{2}{u}\,du$$

$$= 2u - 2\ln|u| + C = \underline{\underline{2(1-\sqrt{x}) - 2\ln|1-\sqrt{x}| + C}}$$

d) Bruker at: $\displaystyle f(b) - f(a) = \int_a^b f'(x)\,dx$

$$\Rightarrow f(1) - f(0) = \int_0^1 f'(x)\,dx = -A_1$$

der A_1 er arealet mellom $f'(x)$ og x-aksen i $[0,1]$

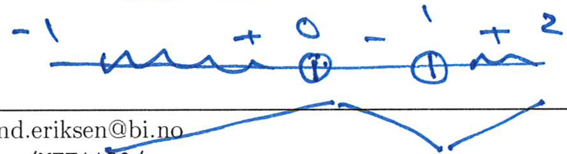
$$A_1 \simeq 4 \text{ ruter} = 4\cdot(1/4)^2 = \frac{4}{16} = \underline{0.25}$$

Konkl: $\displaystyle\int_0^1 f'(x)\,dx = f(1) - f(0) \simeq \underline{\underline{-0.25}}$

e) Stasjonære pkt: $f'(x) = 0$ leser av: $\underline{x=0}$, $\underline{x=1}$

Randpkt: $\underline{x=-1}$, $\underline{x=2}$

$f'(x)$



Maks: Kandidater $\underline{x=0}$, $\underline{x=2}$

$$\int_0^2 f'(x)\,dx = f(2)-f(0) = -A_1 + A_2$$
er positivt siden $A_2 > A_1$

$\Rightarrow f(2) > f(0)$ $\underline{\underline{x=2}}$ er max

Min: $\underline{x=-1}$, $\underline{x=1}$

$$f(1)-f(-1) = \int_{-1}^{1} f'(x)\,dx = A_0 - A_1$$
er positivt siden $A_0 > A_1$

$\Rightarrow f(1) > f(-1)$ $\underline{\underline{x=-1}}$ er min

3. $A = \begin{pmatrix} a & 1 & 2 \\ 1 & a & 1 \\ 2 & 1 & a \end{pmatrix}$

$$|A| = -1\,(a-2) + a\cdot(a^2-4) - 1\,(a-2)$$
$$= -(a-2) + a(a-2)(a+2) - (a-2)$$
$$= (a-2)\cdot(-1 + a(a+2) - 1)$$
$$= (a-2)\cdot(a^2+2a-2)$$

$$-a+2+a^3-4a-a+2 = \underline{a^3-6a+4}$$

a) $\underline{a=0}$: $A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix}$ $|A| = (-2)(-2) = 4 \neq 0$
A^{-1} fins.

$$A^{-1} = \frac{1}{|A|}\cdot \underbrace{\begin{pmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{pmatrix}^T}_{\text{adj}(A)} = \frac{1}{4}\begin{pmatrix} -1 & 2 & 1 \\ 2 & -4 & 2 \\ 1 & 2 & -1 \end{pmatrix}^T = \underline{\frac{1}{4}\begin{pmatrix} -1 & 2 & 1 \\ 2 & -4 & 2 \\ 1 & 2 & -1 \end{pmatrix}}$$

b) $|A| = (a-2)\cdot(a^2+2a-2) = (a-2)\cdot\cancel{(a+3)(a-3)}$

$a^2+2a-2=0$: $a = \dfrac{-2 \pm \sqrt{4+8}}{2} = -1 \pm \dfrac{\sqrt{12}}{2} = -1 \pm \dfrac{\sqrt{4}\cdot\sqrt{3}}{2} = -1 \pm \sqrt{3}$

$|A| = \underline{\underline{(a-2)(a^2+2a-2)}}$

$|A| = 0$: $(a-2)(a^2+2a-2) = 0$

$\underline{a=2}$ eller $a^2+2a-2=0$

$\underline{a = -1 \pm \sqrt{3}}$

<u>Konkl:</u> $|A| = 0$ for

$\underline{a=2}$, $a = \underline{-1+\sqrt{3}}$,

$a = \underline{-1-\sqrt{3}}$

c) Finn $\underline{x} \neq (0,0,0)$ og a slik at $A \cdot \underline{x} = \underline{0}$

$|A| \neq 0$: $A\underline{x} = \underline{0}$ har akkurat én løsn. $\underline{x} = (0,0,0)$

$|A| = 0$: $A\underline{x} = \underline{0}$ har uendelig mange løsn.

$\underline{a=2:}$ $A\underline{x} = \underline{0}$

$$\left(\begin{array}{ccc|c} 2 & 1 & 2 & 0 \\ 1 & 2 & 1 & 0 \\ 2 & 1 & 2 & 0 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 2 & 1 & 2 & 0 \\ 2 & 1 & 2 & 0 \end{array}\right) \begin{array}{l} -2 \\ \\ -2 \end{array}$$

$$\rightarrow \left(\begin{array}{ccc} 1 & 2 & 1 \\ 0 & -3 & 0 \\ 0 & -3 & 0 \end{array}\right) \begin{array}{l} \\ \\ -1 \end{array} \rightarrow \left(\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$x + \cancel{2 \cdot 0} + z = 0 \Rightarrow \underline{x = -z}$

$-3y = 0 \Rightarrow \underline{y = 0}$

z fri

<u>Konkl:</u> vi kan velge $\underline{a=2}$ og $\underline{x} = (x,y,z) = (-z,0,z)$ med $z \neq 0$

for eksempel $\underline{\underline{a=2}}$ $\underline{x} = \underline{\underline{(-1,0,1)}}$

4. $f(x,y) = x^2y - 5xy^2 + xy^3$

a) $f'_x = 2xy - 5y^2 + y^3 = 0 \quad (1) \quad y(2x - 5y + y^2) = 0$

$f'_y = x^2 - 10xy + 3xy^2 = 0 \quad (2) \quad x(x - 10y + 3y^2) = 0$

(1) $y = 0$ eller $2x - 5y + y^2 = 0$

(2) $x = 0$ eller $x - 10y + 3y^2 = 0$

(a) $y=0,\ x=0 \quad \Rightarrow (x,y) = (0,0)$

(b) $y=0,\ x - 10y + 3y^2 = 0 \quad$ — " —

(c) $2x - 5y + y^2 = 0,\ x = 0 \quad -5y + y^2 = 0 \quad y(-5+y) = 0$

$\Downarrow$

$(x,y) = (0,0),\ (0,5)$

(d) $2x - 5y + y^2 = 0,\ x - 10y + 3y^2 = 0$

$\Downarrow$

$x = 10y - 3y^2$

$2(10y - 3y^2) - 5y + y^2 = 0$

$20y - 6y^2 - 5y + y^2 = 0$

$-5y^2 + 15y = 0$

$-5y(y-3) = 0$

$y = 0$ eller $y = 3$

$x = 0 \quad | \quad x = 30 - 27$

$x = 3$

$\Downarrow$

$(x,y) = (0,0),\ (3,3)$

Stasjonære pkt:
$(0,0),\ (0,5),\ (3,3)$

b) $H(f) = \begin{pmatrix} 2y & 2x - 10y + 3y^2 \\ 2x - 10y + 3y^2 & -10x + 6xy \end{pmatrix}$

$H(f)(0,5) = \begin{pmatrix} 10 & 25 \\ 25 & 0 \end{pmatrix} \quad \det = 0 - 25^2 < 0$

$\Rightarrow (0,5)$ sadelpkt

$H(f)(3,3) = \begin{pmatrix} 6 & 3 \\ 3 & 24 \end{pmatrix} \quad \det = 6 \cdot 24 - 9 > 0, \quad \mathrm{tr} = 6 + 24 > 0$

$\Rightarrow (3,3)$ lokalt min

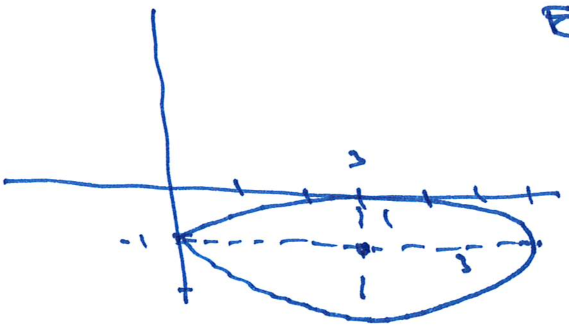
5. a) D: $\underbrace{x^2-6x}_{+9} + 9y^2 + 18y + 9 = 0 \quad +9$

$$(x-3)^2 + 9(y^2+2y+1) = 9$$

$$(x-3)^2 + 9(y+1)^2 = 9 \quad |:9$$

$$\frac{(x-3)^2}{9} + \frac{(y+1)^2}{1} = 1$$

Ellipse, senter $(3,-1)$, halvakser $\underline{a=3}$, $\underline{b=1}$



D er <u>begrenset</u> siden

$$0 \le x \le 6$$
$$-2 \le y \le 0$$

b) max $f = x+3y$ når $x^2-6x+9y^2+18y+9=0$

E.V.setn $\Rightarrow$ det fins et <u>maks</u>

ellipse $\Rightarrow$ det fins ingen tillatte pkt med degenerert bibetingelse.

$$L = x+3y - \lambda(x^2-6x+9y^2+18y+9)$$

$$\boxed{\begin{aligned} L'_x &= 1-\lambda\cdot(2x-6) = 0 \\ L'_y &= 3-\lambda\,(18y+18) = 0 \\ & x^2-6x+9y^2+18y+9=0 \end{aligned}}$$

$\lambda = \dfrac{1}{2x-6}$ $(2x-6 \neq 0)$

$\lambda = \dfrac{3}{18y+18} = \dfrac{1}{6y+6}$ $(18y+18 \neq 0)$

$$\frac{1}{2x-6} = \frac{1}{6y+6} \qquad 6y+6 = 2x-6 \quad |:2$$

$$x-3 = 3y+3 = \underline{3(y+1)}$$

$$\frac{(x-3)^2}{9} + (y+1)^2 = 1$$

$$\frac{(\cancel{3}(y+1))^2}{\cancel{9}\,1} + (y+1)^2 = 1$$

$$(y+1)^2 + (y+1)^2 = 1$$

$$2(y+1)^2 = 1$$
$$(y+1)^2 = 1/2$$
$$y+1 = \pm\sqrt{1/2}$$
$$y = \underline{\pm\sqrt{1/2}-1}$$
$$x-3 = 3(\pm\sqrt{1/2})$$
$$x = \underline{3 \pm 3\sqrt{1/2}}$$

Kandidatpkt:

$$\lambda = \frac{1}{6y+6} = \frac{1}{6}\cdot\frac{1}{y+1}$$
$$= \frac{1}{6}\cdot\frac{1}{\pm\sqrt{1/2}}$$

$(x,y;\lambda) =$

$$\left(3+3\sqrt{1/2},\ \sqrt{1/2}-1;\ \frac{1}{6\cdot\sqrt{1/2}}\right), \leftarrow f = \cancel{3}+3\sqrt{1/2}+3\sqrt{1/2}-\cancel{3} = \underline{6\cdot\sqrt{1/2}}$$

$$\left(3-3\sqrt{1/2},\ -\sqrt{1/2}-1;\ -\frac{1}{6\cdot\sqrt{1/2}}\right) \leftarrow f = \cancel{3}-3\sqrt{1/2} + 3(-\sqrt{1/2}\cancel{-1}) = \underline{-6\sqrt{1/2}}$$

Konkl:

$$f_{max} = \underline{\underline{6\cdot\sqrt{1/2}}} \quad ; \quad \underline{(3+3\sqrt{1/2},\ \sqrt{1/2}-1)}$$

$$= \frac{6\cdot\sqrt{1}\cdot\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}} \text{ med } \lambda = \frac{1}{6\sqrt{1/2}}$$

$$= \frac{6\sqrt{2}}{2} = \underline{\underline{3\sqrt{2}}}$$

6. a) D: $x(x^2+y^2) = x^2 - y^2$

$x^3 + xy^2 = x^2 - y^2$

$\underbrace{x^3 + xy^2 - x^2 + y^2}_{g(x,y)} = \underbrace{0}_{a}$

Degen. bibetingelse:

$g'_x = 3x^2 + y^2 - 2x = 0$ (1)

$g'_y = 2xy + 2y = 0$ (2) $2y(x+1) = 0$

$y = 0$ eller $x = -1$

$3x^2 - 2x = 0$	$3 + y^2 + 2 = 0$
$x(3x-2) = 0$	$y^2 + 5 = 0$
$x = 0$ eller $x = 2/3$	$y^2 = -5$
$(x,y) = (0,0)$, $(2/3, 0)$	Ingen pkt

$g(0,0) = 0$ ok

$g(2/3, 0) = (2/3)^3 - (2/3)^2 \neq 0$

Konkl: (0,0) er eneste tillatte pkt med degenerert bibetingelse

b) $f(x,y) = y$

Nivåkurver for f: $y = c$

(horisontal rett linje)

Kandidatpkt m/ $L'_x = L'_y = 0$, $g(x,y) = 0$

= tillatte pkt med horisontal tangent