MET 1180, forelesning 29, 23. jan. 2024, Runar Ile

Plan
1. Repetisjon – oppgaveregning
2. Arealfunksjonen

1. Repetisjon

subs: $u = 4-x$
$du = -dx$

Oppg. 1 a) $\int \frac{4}{4-x}\,dx = \int \frac{-4}{u}\,du = -4\ln|u| + C$

$$= \underline{\underline{-4\ln|4-x| + C}}$$

b) $\int \frac{4}{4-x^2}\,dx = \int \frac{1}{2-x}\,dx + \int \frac{1}{2+x}\,dx$

2x subst: $u = 2-x$ og $u = 2+x$

$$= \underline{\underline{-\ln|2-x| + \ln|2+x| + C}}$$

Delbrøksopps:

$$\frac{4}{4-x^2} = \frac{A}{2-x} + \frac{B}{2+x}$$

$$= \frac{2A + Ax + 2B - Bx}{(2-x)(2+x)}$$

$$= \frac{(A-B)x + 2(A+B)}{(2-x)(2+x)}$$

$\Downarrow$

$4 = (A-B)x + 2(A+B)$

– for alle x!

så $\begin{cases} A - B = 0 \\ 2(A+B) = 4 \end{cases}$ og

dvs $\begin{cases} A = B \\ A+B = 2 \end{cases}$

dvs $\begin{cases} A = B \\ 2B = 2 \end{cases}$

dvs $\begin{cases} A = 1 \\ B = 1 \end{cases}$

c) $\int \frac{4x}{4-x^2}\,dx = \int \frac{\cancel{4x}^{-2}}{u} \cdot \frac{1}{\cancel{-2x}} \cdot du = \underline{\underline{-2\ln|4-x^2| + C}}$

subst:
$u = 4-x^2$
$du = -2x\,dx$ dvs $dx = \frac{1}{-2x}\,du$

d) $\int \frac{x^2}{4-x^2}\,dx \overset{\text{polyn.}}{\underset{\text{div.}}{=}} \int -1 + \frac{4}{4-x^2}\,dx$

(se b)

$$= \underline{\underline{-x - \ln|2-x| + \ln|2+x| + C}}$$

Oppg 2 d) $\int \frac{x^2-2x+1}{1-x^2}\,dx$

Polynomdiv:

$(x^2-2x+1):(-x^2+1) = -1 + \frac{-2x+2}{1-x^2}$

$-(x^2 \quad -1)$

$-2x+2$

$= \int -1\,dx + \int \frac{-2x+2}{1-x^2}\,dx$

$-2x+2 = 2(1-x)$

$1-x^2 = (1-x)(1+x)$

$= \int -1\,dx + 2\int \frac{1}{1+x}\,dx$

$= \underline{\underline{-x + 2\ln|1+x| + C}}$

f) $\int \frac{2x}{(1-x)^2}\,dx = \int \frac{-2}{1-x} + \frac{2}{(1-x)^2}\,dx$

subst: $u = 1-x$

$du = -dx$

$= 2\ln|1-x| + \int \frac{-2}{u^2}\,du$

$= \underline{\underline{2\ln|1-x| + \frac{2}{1-x} + C}}$

Delbrøksoppspalting

$\frac{2x}{(1-x)^2} = \frac{\overset{=-2}{A}}{(1-x)} + \frac{\overset{=2}{B}}{(1-x)^2}$

$= \frac{-Ax + A + B}{(1-x)^2}$

så $\begin{cases} A + B = 0 \\ -A = 2 \end{cases}$

så $\begin{cases} B = -A = 2 \\ A = -2 \end{cases}$

h) $\int \frac{x^2-2x+1}{(1-x)^2}\,dx$

$= \int 1\,dx = \underline{\underline{x + C}}$

Vil gjøre polynomdiv, men må gange ut nevneren først:

$(1-x)^2 = 1-2x+x^2$

som er lik telleren!

Oppg 3 j) $\int_{-1}^{1} e^x + e^{-x}\,dx$

$-x = u \rightsquigarrow du = -dx$

$\int e^x\,dx + \int e^{-x}\,dx = e^x - e^{-x} + C$

$\int_{-1}^{1} e^x + e^{-x}\,dx = \left[e^x - e^{-x}\right]_{-1}^{1} = e - e^{-1} - (e^{-1} - e^{-(-1)})$

$\underline{\underline{= 2e - 2e^{-1}}}$

2

Start: 9.03

Oppg 4

a) $\int_0^1 xe^x\,dx = \left[(x-1)e^x\right]_0^1 = 0-(-e^0) = \underline{\underline{1}}$

$$\int \overset{u}{\underset{\shortparallel}{x}}\,\overset{v'}{\underset{\shortparallel}{e^x}}\,dx \overset{\text{delvis int.}}{=} \overset{u}{\underset{\shortparallel}{x}}\,\overset{v}{\underset{\shortparallel}{e^x}} - \int \overset{u'}{\underset{\shortparallel}{1}}\cdot \overset{v}{\underset{\shortparallel}{e^x}}\,dx = xe^x - e^x + C = (x-1)e^x + C$$

b) $\int_0^1 x\ln(x^2+1)\,dx = \frac{1}{2}\left[(x^2+1)\ln(x^2+1) - x^2\right]_0^1 = \frac{1}{2}\left[2\ln 2 - 1 - (1\cdot\ln 1 - 1\cdot 0)\right]$

$= \underline{\underline{\ln(2) - \frac{1}{2}}}$

$$\int x\ln(x^2+1)\,dx = \int \cancel{x}\cdot\ln(u)\cdot\frac{1}{2\cancel{x}}\,du$$

$$= \frac{1}{2}\int \ln(u)\,du = \frac{1}{2}\Big(u\ln(u) - u\Big) + C$$

$$= \frac{1}{2}\Big((x^2+1)\ln(x^2+1) - (x^2+1)\Big) + C$$

$$= \frac{1}{2}(x^2+1)\ln(x^2+1) - \frac{1}{2}x^2 - \frac{1}{2} + C$$

$u = x^2+1$

$du = 2x\,dx$

$dx = \frac{1}{2x}\,du$

d) $\int_0^1 \frac{1}{x^2+4x+4}\,dx$

$= \int_0^1 \frac{1}{(x+2)^2}\,dx = \left[-\frac{1}{x+2}\right]_0^1 =$

Prøver delbrøksoppsp. – Må først faktorisere nevner:

$(x+2)^2 = x^2+4x+4$

Ah – deilig: skifterplan til subst. $u = x+2$, $du = dx$

$$\int \frac{1}{u^2}\,du = -\frac{1}{u} + C$$

$$= -\frac{1}{1+2} - \left(-\frac{1}{0+2}\right) = -\frac{1}{3} + \frac{1}{2} = \underline{\underline{\frac{1}{6}}}$$

Oppg 6b) $\int x^3\sqrt{x^2+4}\,dx$

$= \int x^{\cancel{3}\,2}\sqrt{u}\cdot\frac{1}{2\cancel{x}}\,du$

$= \frac{1}{2}\int (u-4)\,u^{\frac{1}{2}}\,du$

$= \frac{1}{2}\int u^{\frac{3}{2}} - 4u^{\frac{1}{2}}\,du$

$= \frac{1}{2}\left(\frac{2}{5}u^{\frac{5}{2}} - \frac{8}{3}u^{\frac{3}{2}}\right) + C$

$= \frac{1}{2}\left(\frac{2}{5}(x^2+4)^{\frac{5}{2}} - \frac{8}{3}(x^2+4)^{\frac{3}{2}}\right) + C$

$= \frac{1}{5}(x^2+4)^{\frac{5}{2}} - \frac{4}{3}(x^2+4)^{\frac{3}{2}} + C$

En mulighet er delvis m. $u = x^2$ og $v' = x\sqrt{x^2+4}$

$u' = 2x \quad v = \frac{1}{3}(x^2+4)^{\frac{3}{2}}$

+ delvis en gang til på: $\int x^2\cdot\frac{1}{3}(x^2+4)^{\frac{3}{2}}\,dx$

Substitusjon: $u = x^2+4$

$du = 2x\,dx$

$dx = \frac{1}{2x}\,du$

og $x^2 = u-4$

Oppg 7c $\int \frac{\sqrt{x}+1}{1-\sqrt{x}}\,dx$

En mulighet:

$u = 1-\sqrt{x}$

$du = -\frac{1}{2\sqrt{x}}\,dx$

$\sqrt{x} = 1-u$

$\sqrt{x}+1 = 2-u$

osv.

Bedre:

$\frac{\sqrt{x}+1}{1-\sqrt{x}}\cdot\frac{(1+\sqrt{x})}{(1+\sqrt{x})} = \frac{x+2\sqrt{x}+1}{1-x}$

$= \frac{x+1}{-x+1} - \frac{2\sqrt{x}}{x-1}$

Setter $u = \sqrt{x}$ så $u^2 = x$

$du = \frac{1}{2\sqrt{x}}\,dx$

så $dx = 2\sqrt{x}\,du$

$\frac{x+1}{-x+1} \overset{\text{poly. div}}{=} -1 + \frac{2}{1-x}$

Da blir $\int \frac{2\sqrt{x}}{x-1}\,dx = \int \frac{2\sqrt{x}\cdot 2\sqrt{x}}{u^2-1}\,du$

$$= 4\int \frac{u^2}{u^2-1}\,du = 4\int 1 + \frac{1}{u^2-1}\,du$$

delbrøksoppspalting

$$= \frac{1/2}{u-1} - \frac{1/2}{u+1}$$

$$= 4u + 2\ln|u-1| - 2\ln|u+1| + C$$

Dette gir

$$\int \frac{\sqrt{x}+1}{1-\sqrt{x}}\,dx = \int -1 + \frac{2}{1-x}\,dx - \left(4\sqrt{x} + 2\ln|\sqrt{x}-1| - 2\ln|\sqrt{x}+1|\right) + C$$

$$= -x - 2\ln|1-x| - 4\sqrt{x} - 2\ln|\sqrt{x}-1| + 2\ln|\sqrt{x}+1| + C$$

$$= \underline{\underline{-x - 4\sqrt{x} - 4\ln|1-\sqrt{x}| + C}}$$