| | Emne | Lærebok | Oppgaver |
|---|---|---|---|
| 1 | Delbrøkoppspaltning | [E] 5.5 | 5.5.1 - 5.5.6 |
| 2 | Bestemte integral | [E] 5.6 | 5.6.1 - 5.6.2 |

## ① Delbrøksoppspaltning

Integrasjon av rasjonale uttrykk: $\int \frac{p(x)}{q(x)}\,dx$

polynom

polynom

(i) Bruk polynomdivisjon hvis det kan forenkle uttrykket

(hvis graden til teller $\geq$ grad til nevner)

$$\frac{p(x)}{q(x)} = p(x) : q(x) \rightsquigarrow \frac{p(x)}{q(x)} = \text{polynom} + \frac{\text{rest}}{q(x)}$$

grad < grad nevner

Eks: $x^3 : x^2 - 2x = x + 2$

$-(x^3 - 2x^2)$

$2x^2$

$-(2x^2 - 4x)$

$4x \leftarrow$ rest

$$\frac{x^3}{x^2-2x} = x + 2 + \frac{4x}{x^2-2x}$$

$$= x + 2 + \frac{4}{x-2}$$

(ii) Graden til nevner = 1 : Substitusjon u = nevner

Eks: $\int \frac{2}{4-x}\,dx = \int \frac{2}{u}\frac{1}{(-1)}\,du = -2\ln|u| + C$

$\boxed{\begin{array}{l} u = 4-x \\ du = -1\cdot dx \end{array}}$

$= -2\ln|4-x| + C$

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(iii) Bruke substitusjon (for eks. u = nevner)

Eks: $\int \frac{x}{4-x^2}\,dx = \int \frac{\cancel{x}}{u}\frac{1}{(-2\cancel{x})}\,du = -\frac{1}{2}\int \frac{1}{u}\,du$

$\boxed{\begin{array}{l} u = 4-x^2 \\ du = -2x\,dx \end{array}} \Rightarrow dx = \frac{1}{-2x}\cdot du$

$= -\frac{1}{2}\ln|u| + C = \underline{-\frac{1}{2}\ln|4-x^2| + C}$

iv) Delbrøksoppspalting

Eks: $\int \frac{2}{4-x^2}\,dx$

~~$\boxed{\begin{array}{l} u = 4-x^2 \\ du = -2x\,dx \end{array}} = \int \frac{\cancel{2}}{u}\frac{1}{(-\cancel{2}x)}\,du = -\int \frac{1}{ux}\,du$~~

Delbrøksoppspalting:

- faktoriserer nevner:

$4-x^2 = (2+x)(2-x)$

Alt: $4-x^2 = 0$, $x^2 = 4$, $x = \pm 2$; $4-x^2 = -(x-2)(x+2) = (2-x)(x+2)$

- skriv om brøken:

$\frac{2}{(2+x)(2-x)} = \frac{A}{2+x} + \frac{B}{2-x} \quad \Big| \cdot (2+x)(2-x)$ $\quad A = 1/2,\ B = 1/2$

Konklusjon: $\frac{2}{4-x^2} = \frac{1/2}{2+x} + \frac{1/2}{2-x}$

- bestem A og B

$2 = A(2-x) + B(2+x)$

Setter inn: $x=2$, $x=-2$

$2 = \cancel{A\cdot 0} + B\cdot 4$

$2 = A\cdot 4 + \cancel{B\cdot 0}$

$\boxed{\begin{array}{l} B = 1/2 \\ A = 1/2 \end{array}}$

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Altså:

$$\int \frac{2}{4-x^2}\,dx = \int \frac{1/2}{2+x} + \frac{1/2}{2-x}\,dx$$

$$= 1/2 \cdot \ln|2+x| + \frac{1/2}{-1}\ln|2-x| + C$$

$$= \frac{1}{2}\ln|2+x| - \frac{1}{2}\ln|2-x| + C$$

$$\int \frac{1/2}{2-x}\,dx = \int \frac{1/2}{u}\frac{1}{(-1)}\,du$$

$$\boxed{\begin{array}{l} u = 2-x \\ du = -1 \cdot dx \end{array}} \Rightarrow dx = \frac{1}{(-1)}du$$

$$= \frac{1/2}{-1}\int \frac{1}{u}\,du = -\frac{1}{2}\ln|u| + C$$

$$= -\frac{1}{2}\ln|2-x| + C$$

Eks: $\int \frac{x}{1-2x+x^2}\,dx$

$1-2x+x^2 = (1-x)^2$ eller

$$1-2x+x^2 = 0$$

$$x^2-2x+1 = 0$$

$$x = \frac{2 \pm \sqrt{(-2)^2-4\cdot1\cdot1}}{2\cdot1} = \frac{2\pm 0}{2}$$

$x_1 = 1$, $x_2 = 1$

$$1-2x+x^2 = (x-1)^2$$

$$\frac{x}{(1-x)^2} = \frac{A^{=-1}}{1-x} + \frac{B^{=1}}{(1-x)^2} \quad | \cdot (1-x)^2$$

$$x = A\cdot(1-x) + B$$

$x=1$: $1 = \cancel{A\cdot 0} + B$ $\quad B=1$

$x=0$: $0 = A\cdot 1 + B$ $\quad A = -B = -1$

$$\int \frac{x}{1-2x+x^2}\,dx = \int \frac{-1}{1-x} + \frac{1}{(1-x)^2}\,dx = \frac{-1}{-1}\ln|1-x| + \int \frac{1}{(1-x)^2}\,dx$$

$$= \ln|1-x| + \int \frac{1}{u^2}\frac{1}{(-1)}\,du = \ln|1-x| - \int u^{-2}\,du$$

$$\boxed{\begin{array}{l} u = 1-x \\ du = -1\,dx \end{array}}$$

$$= \ln|1-x| - \left(\frac{u^{-1}}{-1}\right) + C = \ln|1-x| + \frac{1}{u} + C$$

$$= \ln|1-x| + \frac{1}{1-x} + C$$

**Eks:** $\int \frac{1}{x^2+1}\,dx = \arctan(x) + C$

nevner kan ikke faktoriseres i lineære uttrykk

omvendt funksjon til $\tan x = \frac{\sin x}{\cos x}$

**Eks:** $\int \frac{12}{x^3 - x}\,dx = \int \frac{-12}{x} + \frac{6}{x-1} + \frac{6}{x+1}\,dx$

$x^3 - x = x(x^2-1)$

$= x(x-1)(x+1)$

$$\frac{12}{x(x-1)(x+1)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1} \quad | \cdot x(x-1)\cdot(x+1)$$

$$12 = A(x^2-1) + B(x^2+x) + C(x^2-x)$$

$$= Ax^2 + Bx^2 + Cx^2 + Bx - Cx - A$$

$$0\cdot x^2 + 0\cdot x + 12 = \underbrace{(A+B+C)}_{=0}x^2 + \underbrace{(B-C)}_{=0}x + \underbrace{(-A)}_{=+12}$$

$A+B+C = 0$ $\quad -12 + 2C = 0$ $\quad \frac{2C}{2} = \frac{12}{2}$

$B - C = 0$ $\quad B = C$ $\quad C = 6$

$-A = 12$ $\quad A = -12$ $\quad B = 6$

$$= -12\ln|x| + 6\ln|x-1| + 6\ln|x+1| + C$$

## (2) Bestemte integral

$$[F(x)]_a^b$$

$$\int_a^b f(x)\,dx = [F(x)+C]_a^b = F(b)-F(a)$$

$$= (F(b)+C)-(F(a)+C) = F(b)-F(a)$$

Husk: Det bestemte integralet gir et tall som svar.

Husk: $\int_a^b f(x)\,dx$ er definert hvis $f(x)$ er kont. på $[a,b]$

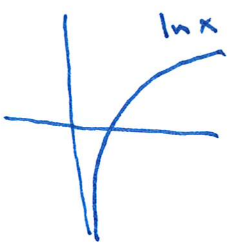
Eks: $\int_0^1 x^2-3\,dx = \left[\frac{1}{3}x^3-3x\right]_0^1 = \left(\frac{1}{3}\cdot 1^3-3\cdot 1\right)-\left(\frac{1}{3}\cdot 0^3-3\cdot 0\right)$

$= \frac{1}{3}-3-0 = \frac{1}{3}-\frac{9}{3}$

$= \underline{\underline{-8/3}} = \underline{\underline{-2.66..}}$

Eks: $\int_0^1 \frac{2}{4-x^2}\,dx = \left[\frac{1}{2}\ln|2+x| - \frac{1}{2}\ln|2-x|\right]_0^1$

$= \left(\frac{1}{2}\ln 3 - \frac{1}{2}\underbrace{\ln 1}_{=0}\right) - \left(\cancel{\frac{1}{2}\ln 2} - \cancel{\frac{1}{2}\ln 2}\right) = \underline{\underline{\frac{1}{2}\ln 3}}$




$\int_1^2 \frac{2}{4-x^2}\,dx = \left[\frac{1}{2}\ln|2+x| - \frac{1}{2}\ln|2-x|\right]_1^2$

$= \left(\frac{1}{2}\ln 4 - \frac{1}{2}\ln 0\right) - \left(\frac{1}{2}\ln 3 - \frac{1}{2}\ln 1\right) = \infty$

$$\int_1^3 \frac{2}{4-x^2}\,dx = \left[\frac{1}{2}\ln|2+x| - \frac{1}{2}\ln|2-x|\right]_1^3$$

$$= \left(\frac{1}{2}\ln 5 - \cancel{\frac{1}{2}\ln 1}\right) - \left(\frac{1}{2}\ln 3 - \cancel{\frac{1}{2}\ln 1}\right) = \underline{\frac{1}{2}\ln 5 - \frac{1}{2}\ln 3}$$