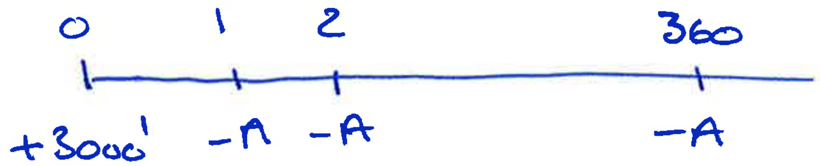
Løsning: Øvingsoppgaver Oppgavesett I

MET1180
Sep 2017

1.



$r = \frac{2{,}10\%}{12} = 0{,}00175$

a)

$$L - \frac{A}{1+r} \cdot \frac{1-\left(\frac{1}{1+r}\right)^{360}}{1-\frac{1}{1+r}} = 0$$

$$L = A \cdot \frac{(1+r)^{360}-1}{r\cdot(1+r)^{360}} \Rightarrow A = L \cdot \frac{r\cdot(1+r)^{360}}{(1+r)^{360}-1}$$

$$= 3.000.000 \cdot \frac{0{,}00175 \cdot 1{,}00175^{360}}{1{,}00175^{360}-1}$$

$$A \simeq \underline{11.239.21 \text{ kr}}$$

Samlede renter: $360 \cdot A - L \simeq 360 \cdot 11{,}239{,}21 \text{ kr} - 3.000.000 \text{ kr}$

$\simeq \underline{1.046.113{,}98 \text{ kr}}$

Dersom terminbeløpet A avrundes slik at A = 11,239,21 kr, blir samlede renter $\underline{1.046.115{,}60 \text{ kr}}$.

b)

Alt A: Totalt 11.239,21 kr + 30 kr = $\underline{11.269{,}21 \text{ kr}}$ / mnd.

Alt B: Terminbeløp $A' = \underline{11.330.15 \text{ kr}}$ / mnd

Alt A lønner seg

$$A' = L \cdot \frac{r' \cdot (1+r')^{360}}{(1+r')^{360}-1} \simeq 11.330{,}15 \text{ kr}$$

med $r' = \frac{2{,}16\%}{12} = 0{,}0018$

c) Alt B:

Effektiv rente $\left(1+\frac{2,16\%}{12}\right)^{12}-1 = 1,0018^{12}-1 \simeq \underline{2,18\%}$

Alt A:

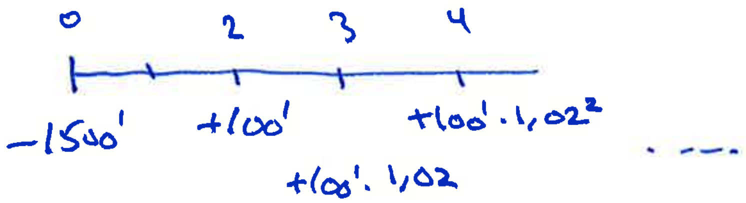
Vet at eff. rente er mindre enn 2,18% siden Alt A lønner seg (her mindre mnd-beløp). Samtidig er effektiv rente uten termingebyr $\left(1+\frac{2,10\%}{12}\right)^{12}-1 = 1,00175^{12}-1 \simeq 2,12\%$.
Derfor kan vi si at med gebyrer er effektiv rente i intervallet

$$\boxed{2,12\% < r_{eff} < 2,18\%}$$

(Man kan finne effektiv rente $r_{eff} \simeq 2,14\%$ ved å bruk Excel eller andre hjelpemidler).

BI-kalk: 3.000.000 $\boxed{PV}$ −11269,21 $\boxed{PMT}$ 360 $\boxed{N}$
$\boxed{I/YR}$ ~~[illegible]~~ ÷100 =
÷12+1 = $\boxed{y^x}$ 12 = −1 = ← 0,0214 ≃ $\underline{\underline{2,14\%}}$

2.



r = 0,10

a) Nåverdi: $-1500.000 + \frac{100.000}{1,10^2} + \frac{100.000\cdot 1,02}{1,10^3} + \dots$

$= -1.500.000 + \frac{100.000}{1,10^2}\cdot\frac{1}{1-\frac{1,02}{1,10}}$

uendelig geom. rekke med $k=\frac{1,02}{1,10}<1$

$= -1.500.000 + \frac{100.000}{1,10\cdot 0,08}$

$\simeq \underline{-363.636 \text{ kr}}$

Negativ nåverdi: Investeringen lønner seg ikke med r=10%.

b) Vekst: g (isteden for vekst 2%)

Nåverdi: $-1.500.000 + \frac{100.000}{1,10^2}\cdot\frac{1}{1-\frac{1+g}{1,10}} = 0$

$= -1.500000 + \frac{100.000}{1,10\cdot(0,10-g)} = 0$

setter nåverdi lik null for å finne ut ved hvilke vekst g investeringen slutter å være lønnsom

$\frac{100.000}{1,10\cdot(0,10-g)} = 1.500.000$

$\frac{1}{0,10-g} = \frac{1.500.000}{100000}\cdot 1,10$

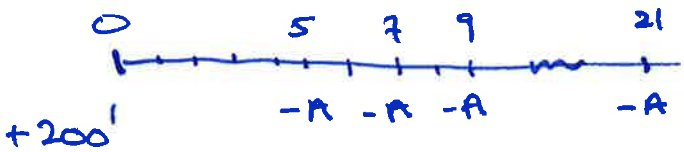
$\frac{1}{0,10-g} = 15\cdot 1,10 = 16,5$

$0,10-g = \frac{1}{16,5}$

$g = 0,10 - \frac{1}{16,5} \simeq \underline{3,94\%}$

Investeringen er lønnsom om tilbakebetalingen vokser med minst $\underline{3,94\%}$ per år.

3.



$r = \frac{18\%}{12} = 0,015$

a) Finner først A: $200.000 - \frac{A}{(1+r)^5} - \frac{A}{(1+r)^7} - \dots - \frac{A}{(1+r)^{21}} = 0$

$200.000 = \frac{A}{(1+r)^5} + \dots + \frac{A}{(1+r)^{21}}$

$= \frac{A}{(1+r)^5}\cdot\frac{1-\left(\frac{1}{(1+r)^2}\right)^9}{1-\frac{1}{(1+r)^2}}$

geom. rekke med $k = \frac{1}{(1+r)^2}$ og $\frac{21-5}{2}+1 = 9$ ledd

$$200.000 = \frac{A \cdot \left((1+r)^{18}-1\right)}{(1+r)^{18} \cdot (1+r)^3 \cdot \left((1+r)^2-1\right)} = A \cdot \frac{(1+r)^{18}-1}{(1+r)^{21} \cdot (r^2+2r)}$$

$$A = 200.000 \cdot \frac{(1+r)^{21} \cdot r(r+2)}{(1+r)^{18}-1} \simeq \underline{26.888,29 \text{ kr}}$$

Samlede renter: $9 \cdot A - 200.000 \simeq \underline{\underline{41.994,62 \text{ kr}}}$

(Bruker vi avrundet $A = 26.888,29$ kr, blir samlede renter
$9 \cdot A - 200.000 \simeq 41.994,61$ kr)

b) $\underline{r' = 6\%/12 = 0,005}$:

$$A' = 200.000 \cdot \frac{(1+r')^{21}\, r'(r'+2)}{(1+r')^{18}-1} \simeq 23.702,94$$

Samlede renter: $9A' - 200.000 \simeq \underline{13.326,48 \text{ kr}}$

Renter: Redusert til $18\%/3 = 6\%$, dvs $1/3$ av r.
Samlede renter er mindre enn $1/3$ av opprinnelig samlede renter. Dette må skyldes at lånet betales raskere tilbake med $r' = 6\%$.

4.

a) $\frac{7}{x} = \frac{8}{x+1} \quad \Big| \cdot x(x+1)$

$7(x+1) = 8x$

$7x + 7 = 8x$

$\underline{\underline{x = 7}}$

b) $x^5 = x$

$x^5 - x = 0$

$x(x^4 - 1) = 0$

$\underline{x = 0}$ eller $x^4 = 1$

$x^2 = \pm 1$

$x^2 = 1$

$\underline{x = \pm 1}$

$\underline{\underline{x = 0,\ x = 1,\ x = -1}}$

d) $\sqrt{3x} - 4 = \sqrt{3x-4}$

$(\sqrt{3x} - 4)^2 = 3x - 4$

$\cancel{3x} - 8\sqrt{3x} + 16 = \cancel{3x} - 4$

$20 = 8\sqrt{3x}$

$\sqrt{3x} = 20/8 = 10/4 = 5/2$

$(\sqrt{3x})^2 = (5/2)^2 = 25/4$

$3x = 25/4$

$x = \underline{25/12}$

VS: $\sqrt{3 \cdot 25/12} - 4 = 5/2 - 4 = -3/2$

HS: $\sqrt{3 \cdot 25/12 - 4} = \sqrt{\frac{25}{4} - 4} = \sqrt{\frac{9}{4}} = 3/2$

ingen løsning

[illegible]) $x^3 - 2x + 1 = 0$

$x = 1$: løsn. siden $1^3 - 2 \cdot 1 + 1 = 0$

$x^3 - 2x + 1 = 0$

$(x-1)(x^2 + x - 1) = 0$

$x = 1$ eller $x^2 + x - 1 = 0$

$x = \frac{-1 \pm \sqrt{5}}{2}$

$x = 1,\ x = \frac{-1+\sqrt{5}}{2},\ x = \frac{-1-\sqrt{5}}{2}$

$x^3 - 2x + 1 : x - 1 = x^2 + x - 1$

$-(x^3 - x^2)$

$x^2 - 2x + 1$

$-(x^2 - x)$

$-x + 1$

$-(-x + 1)$

0

c) $x^6 + x^4 = 2x^2$

$x^6 + x^4 - 2x^2 = 0$

$x^2(x^4 + x^2 - 2) = 0$

$x = 0$ eller $x^4 + x^2 - 2 = 0$ $\quad u = x^2$

$u^2 + u - 2 = 0$

$u = \frac{-1 \pm \sqrt{1+8}}{2} = \frac{-1 \pm 3}{2}$

$u = 1,\ u = -2$

$x^2 = 1$ eller $\cancel{x^2 = -2}$

$x = \pm 1$

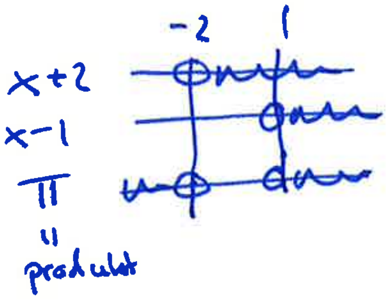
$x = 0,\ x = 1,\ x = -1$

5.

a) $x^2 + x < 2$

$x^2 + x - 2 < 0$

$(x+2)(x-1) < 0$



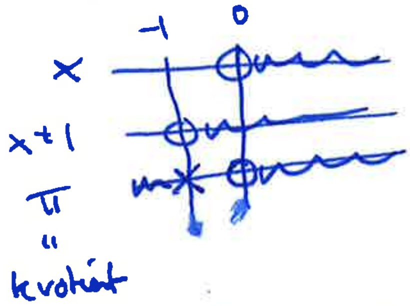
Løsn: $\underline{\underline{(-2, 1)}}$

b) $\frac{1}{x+1} \geq 1$

$\frac{1}{x+1} - \frac{x+1}{x+1} \geq 0$

$\frac{-x}{x+1} \geq 0$

$\frac{x}{x+1} \leq 0$



Løsn: $\underline{\underline{(-1, 0]}}$

c) $\frac{5x+2}{x^2+7x-8} < 1$

$\frac{5x+2-(x^2+7x-8)}{x^2+7x-8} < 0$

$\frac{-x^2-2x+10}{(x+8)(x-1)} < 0$

$\frac{-(x-x_1)(x-x_2)}{(x+8)(x-1)} < 0$

$-x^2 - 2x + 10 = 0$

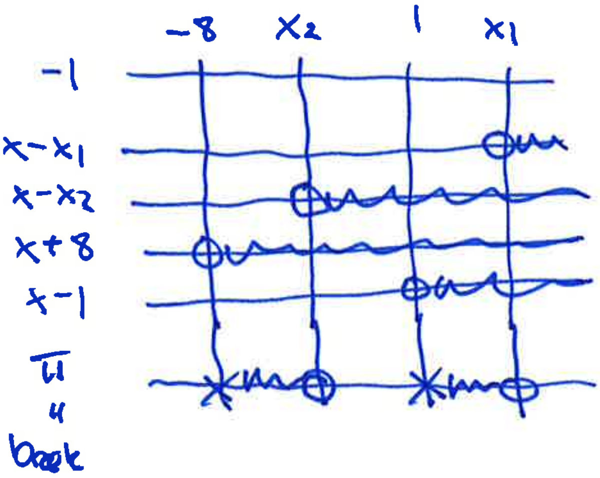
$x = \frac{2 \pm \sqrt{4+40}}{-2} = -1 \pm \frac{\sqrt{44}}{2}$

$= -1 \pm \sqrt{11}$

$x_1 = -1 + \sqrt{11} \simeq 2{,}32$

$x_2 = -1 - \sqrt{11} \simeq -4{,}32$

$-x^2 - 2x + 10 = -(x-x_1)(x-x_2)$



Løsning: $x > x_1$ eller $x_2 < x < 1$ eller $x < -8$

$\underline{\underline{(\leftarrow, -8) \cup (x_2, 1) \cup (x_1, \rightarrow)}}$

med $x_1 = -1 + \sqrt{11} \simeq 2{,}32$

$x_2 = -1 - \sqrt{11} \simeq -4{,}32$

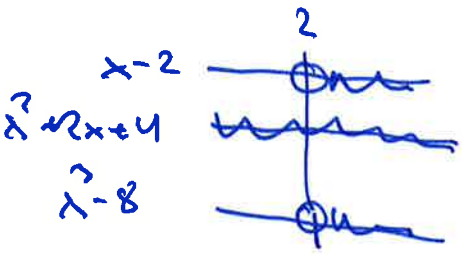
d) $x^3 > 8$

$x^3 - 8 > 0$

$(x-2)(x^2+2x+4) > 0$

$x^3 - 8 : x - 2 = x^2 + 2x + 4$
$-(x^3 - 2x^2)$
$2x^2 - 8$
$-(2x^2 - 4x)$
$4x - 8$
$4x - 8$
0

(siden $x = 2$ er løsn. av $x^3 = 8$)



Løsn: $x > 2$

$(2, \rightarrow)$

$x^2 + 2x + 4 = 0$:

$x = \frac{-2 \pm \sqrt{4-16}}{2}$ ingen løsn.

$\Downarrow$

$x^2 + 2x + 4 > 0$ for alle x

e) $x^2 - x^4 + 7 < 5$

$-x^4 + x^2 + 2 < 0 \quad | \cdot (-1)$

$x^4 - x^2 - 2 > 0$

$(x^2 - 2)(x^2 + 1) > 0$

$(x - \sqrt{2})(x + \sqrt{2})(x^2 + 1) > 0$ ← alltid pos.

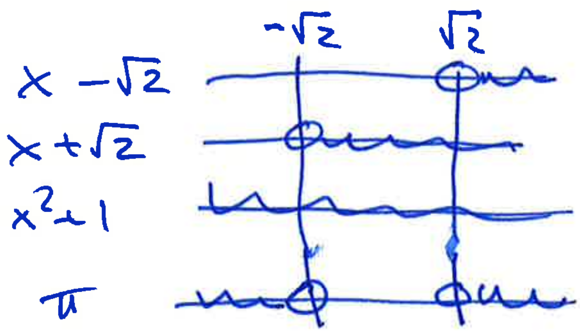
$u = x^2$: $u^2 - u - 2 = 0$

$u = \frac{1 \pm \sqrt{1+8}}{2}$

$u = \frac{1 \pm 3}{2} = 2, -1$

$x^2 = 2$ eller $\cancel{x^2 = -1}$

$x = \pm\sqrt{2}$



Løsn: $x > \sqrt{2}$ eller $x < -\sqrt{2}$

$(\leftarrow, -\sqrt{2}) \cup (\sqrt{2}, \rightarrow)$

6.

$$\frac{6}{x+1} - 1 = \frac{3}{x} + \frac{3}{x+2} \quad \Big| \cdot x(x+1)(x+2)$$

$$6x(x+2) - x(x+1)(x+2) = 3(x+1)(x+2) + 3x(x+1)$$

$$6x^2 + 12x - x(x^2+3x+2) = 3(x^2+3x+2) + 3x^2 + 3x$$

$$-x^3 - 3x^2 - 2x = 6$$

$$x^3 + 3x^2 + 2x + 6 = 0$$

Mulige heltallsløsn:

$x = \pm 1, \pm 2, \pm 3, \pm 6$

$x = \pm 1, \pm 2$ går ikke

$x = -3$ er løsn.

$$(x+3)(x^2+2) = 0$$

$x = -3$ eller $x^2 = -2$

$$\underline{\underline{x = -3}}$$

$$\boxed{\begin{aligned} &x^3 + 3x^2 + 2x + 6 : x+3 = x^2+2 \\ &\underline{x^3 + 3x^2} \\ &\qquad 2x+6 \\ &\qquad \underline{2x+6} \\ &\qquad\quad 0 \end{aligned}}$$