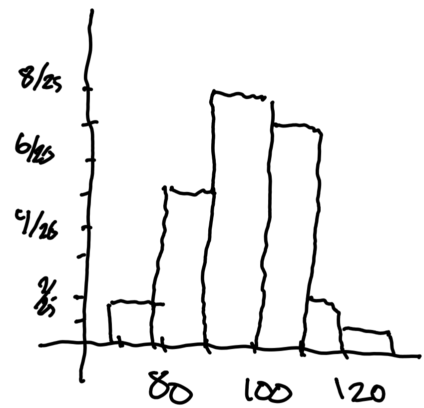
Løsning: Oppgavesett 10

1.

a) **Frekvenstabell:**

71 - 80	2	2/25
81 - 90	5	5/25
91 - 100	8	8/25
101 - 110	7	7/25
111 - 120	2	2/25
121 - 130	1	1/25

Histogram:



ja, det likner på en normalfordeling

b)

Gjennomsnitt:

kalk

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{1}{25}(104 + \dots + 97) \simeq \underline{98.04}$$

Median: 79 80 83 86 87 87 88 91 92 94 96 97
98 98 99 103 104 104 107 108 109 109 111
119 122 $\Rightarrow$ Median = $\underline{98}$

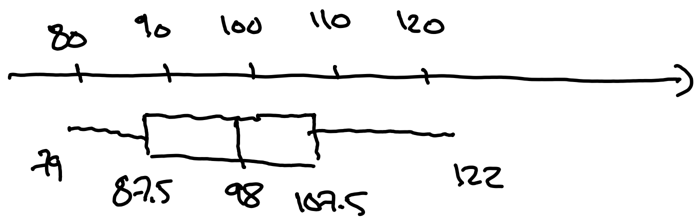
Siden fordelingen er omtrent som normalfordeling, forventer vi at gjennomsnitt og median er omtrent like. Differansen er

$$98{,}04 - 98 = \underline{\underline{0{,}04}}$$

c) Øvre kvartil: $\underline{\underline{107.5}}$

Nedre kvartil: $\underline{\underline{87.5}}$ (gjennomsnitt av 6. og 7. største/minste måling)

$$\frac{25+1}{4} = \underline{6.5}$$



Boksplott:

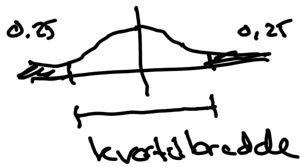
d) Kvartilbredde: $107.5 - 87.5 = \underline{\underline{20}}$

Standardavvik: $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$

$$= \frac{1}{24}\left((104-98.04)^2 + \dots + (97-98.04)^2\right) \overset{\text{kalk.}}{\approx} 133.7$$

$$\Rightarrow s \approx \sqrt{133.7} \simeq \underline{\underline{11.6}}$$

Ettersom fordelingen likner normalfordeling,
og vi har at for std. normalfordeling,



så er kvartilbredden

$z_{0.25} - z_{0.75} \simeq 1.35$

så forventer vi kvartilbredde $\simeq 1.35 s$

Kvartilbredde: $\underline{20}$

$1.35 \cdot s = 1.35 \cdot 11.6 = \underline{15.6}$

$\underline{2}$. Konfidensnivå α : $(1-\alpha) \cdot 100\%$ konf. intervall
er $\left(\bar{x} - z_{\alpha/2} \cdot \sigma/\sqrt{n},\ \bar{x} + z_{\alpha/2}\, \sigma/\sqrt{n}\right)$

a) $\underline{\alpha = 0.06}$: $\bar{x} = 98.04$ $n = 25$
$\sigma = 10$ $z_{\alpha/2} = 1.96$ kalk

$\Downarrow$

$$\left(\bar{x} - z_{\alpha/2} \cdot \sigma/\sqrt{n},\ \bar{x} + z_{\alpha/2} \cdot \sigma/\sqrt{n}\right) = 98.04 \pm 3.92$$

$$= (94.12,\ 101.96) = \underline{\underline{(94.1,\ 102.0)}}$$

b) Som a) med $\alpha = 0.10$

$z_{\alpha/2} = 1.645$ ← (kalk)

$$\left(\bar{x} - z_{\alpha/2} \cdot \sigma/\sqrt{n}\,,\ \bar{x} + z_{\alpha/2} \cdot \sigma/\sqrt{n}\right)$$

$$= 98.04 \pm 3.29 = (94.75,\ 101.33)$$

$$= \underline{\underline{(94.8\ ,\ 101.3)}}$$

c) $P(\mu > B) = 0.02$

$\Updownarrow$

$P(\mu \leq B) = 1 - 0.02 = 0.98$

↑

Ensidig konfidensnivå > 0, $\alpha = 0.02$

$\Rightarrow$ Bruker $z_\alpha = z_{0,02} = 2.054$

$$B = \bar{x} + z_\alpha \cdot \sigma/\sqrt{n} = 98.0 + 2.054 \cdot 10/\sqrt{25}$$

$$= 98.0 + 4.1 = \underline{\underline{102.1}}$$

3. Ukjent σ: Bruker s^2 for å estimere σ^2

$s^2 \simeq 133.7 \Rightarrow s \simeq 11.56 \simeq \underline{11.6}$ ← (fra ldg)

Bruker t-fordeling med $n-1 = 24$ frih.gr. isteden for std. normalfordeling:

$$\left(\bar{x} - t^{n-1}_{\alpha/2} \cdot s/\sqrt{n}\,,\ \bar{x} + t^{n-1}_{\alpha/2} \cdot s/\sqrt{n}\right)$$

a) $\bar{x} = 98.04$

$S = 11.56$

$n = 25$

$\alpha = 0.05$

$\Downarrow$

$t_{\alpha/2}^{n-1} = t_{0,025}^{24} \stackrel{\sim}{=} 2.064$

kalk: 24 $\boxed{\text{INV}}$

$\boxed{\text{df}, t \rightleftarrows p}$ 0.975 $\boxed{=}$

Skal gi 2,064

Hvis kalkulatoren ikke klarer dette, betyr det at batteriet er dårlig.

$$\bar{X} \pm t_{\alpha/2}^{n-1} \cdot s/\sqrt{n} = 98.04 \pm 2.064 \cdot \frac{11.56}{\sqrt{25}}$$

$$= 98.04 \pm 4.77 = \underline{\underline{(93.3\ ,\ 102.8)}}$$

b) Som i a) men $\alpha = 0.10 \Rightarrow t_{\alpha/2}$ ✓ kalk

$= t_{0.05}^{24} \simeq 1.711$

$$\bar{x} \pm t_{\alpha/2}^{n-1} \cdot s/\sqrt{n} = 98.04 \pm 1.711 \cdot \frac{11.56}{\sqrt{25}}$$

$$= 98.04 \pm 3.96 = \underline{\underline{(94.1\ ,\ 102.0)}}$$

c) $P(\mu > B) = 0.02$

$\Updownarrow$

$P(\mu \le B) = 1 - 0.02 = 0.98$

$\uparrow$

Ensidig konfidensintervall, $\alpha = 0.02$

$t_\alpha^{n-1} = t_{0.02}^{24} \cong \underline{2.172}$

$$B = \bar{x} + t_\alpha^{n-1} \cdot s/\sqrt{n}$$

$$= 98.04 + 2.172 \cdot 11.56/\sqrt{25}$$

$$\cong \underline{\underline{103.1}}$$

<u>4.</u> $\bar{x} \pm t_{0.025}^{4} \cdot s/\sqrt{5} = 98.04 \pm 2.776 \cdot s/\sqrt{5}$

hvor $\bar{x}, s$ regnes ut fra hver gruppe av fem observasjoner $\leftarrow$ Bruk kalk.

a) $[91.8, 116.2]$

b) $[74.3, 123.7]$

c) $[91.3, 107.5]$

d) $[86.5, 109.5]$

e) $[81.1, 98.6]$

5. Se løsning for Eksamen MET1190 12/2019, Oppg. 2