

Løsning: Oppgavesett 15

1. Se Oppgavesett 14.

2. X = antall blå kugler
 Y = antall røde kugler

a) V = de vinner: $X > Y$

$$\begin{aligned} p(V) &= p(X > Y) \\ &= \frac{5+4+3+2+1}{36} \\ &= \frac{15}{36} = \frac{5}{12} \end{aligned}$$



b) Antar binomisk forsøk
(uavhengighet)

W = antall ganger den vinner
på 50 kugler

$$W \sim \text{Binom}(n, p) \quad \begin{aligned} n &= 50 \\ p &= p(V) = 5/12 \end{aligned}$$

$$\begin{aligned} p(W \geq 20) &= 1 - p(W \leq 19) \\ &\approx 1 - p\left(z \leq \frac{19 + 0.5 - \mu}{\sigma}\right) \end{aligned}$$

brukes normal-
tilnærming
med
helhetskor.
(se neste
side)

W : Binom (n, p) $n=50$
 $p=5/12$

$$\mu = E(W) = np = 50 \cdot 5/12 = \frac{250}{12} \approx 20.83$$

$$\sigma^2 = \text{Var}(W) = npq = 50 \cdot 5/12 \cdot 7/12$$
$$= \frac{250}{12} \cdot \frac{7}{12} = \frac{1750}{144} \approx 12.2$$

$n=50$ "stor" $\Rightarrow$ normaltilnærning
($npq \geq 5$) er god
(spesielt med
helhetstestkorr.)

$$P(W \geq 20) = 1 - P(W \leq 19)$$
$$\approx 1 - P\left(Z \leq \frac{19 + 0.5 - 20.83}{\sqrt{12.2}}\right)$$
$$\approx 1 - \Phi(-0.382)$$
$$\approx 1 - 0.351 \approx \underline{\underline{0.649}}$$

c) Hvis du vinner hver gang:

2000 $\rightarrow$ 4000 $\rightarrow$ 8000 $\rightarrow$ 16000 $\rightarrow$ 32000

✓ vinner fire
 ganger pr. rad:

$$32.000 - 2000 = 30.000$$

✓ ellers : - 2000

$$p \text{ (vinner 4 ganger p\acute{e} red)} \\ = (5/12)^4$$

Sannsynlighet for \u00e5 tape alt:

$$1 - (5/12)^4 \approx 1 - 0.03 = \underline{\underline{0.97}}$$

F\u00f8rreket totalgevinst:

$$+ 30.000 \cdot (5/12)^4 - 2000 \cdot (1 - (5/12)^4) \\ \approx 30000 \cdot 0.03 - 2000 \cdot 0.97 \approx \underline{\underline{-1.035 \text{ kr}}}$$

3.

a) Hypotesetest: andelen p red \u00e5 eller 6

$$H_0: p \geq 1/3 = p_0$$

$$\alpha = 0.10$$

$$H_1: p < 1/3 = p_0$$

$$T = \frac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/n}}$$

F\u00f8rkastingsomr\u00e5de: $T < -Z_\alpha = -1.28$

Relevant verdi:

$$\hat{p} = 4/25 = 0.16 \Rightarrow$$

$$T = \frac{0.16 - 1/3}{\sqrt{1/3 \cdot 2/3 / 25}} \\ \approx \underline{\underline{-1.84}}$$

Vi f\u00f8rkyler H_0 : sannsynligheten er mindre enn $1/3$.

$$b) H_0: \mu = 3.5 = \mu_0$$

$$H_1: \mu \neq 3.5 = \mu_0$$

$$T = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{\bar{x} - 3.5}{s/\sqrt{25}}$$

Realisiert verdi:

$$\left. \begin{array}{l} \bar{x} = 3.28 \\ s \hat{=} 1.568 \end{array} \right\} T = \frac{3.28 - 3.5}{1.568/\sqrt{25}} \approx -0.70$$

$$p\text{-verdi: } p(T > 0.70)$$

$$= 2 \cdot p(T < -0.70)$$

$$= 2 \cdot 0.245 = \underline{\underline{0.49}}$$

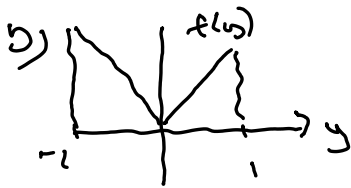
$$c) p\text{-verdi: } p = 0.043 < 0.05 = \alpha$$

$\Rightarrow$ Vi forhaster H_0

(siden realisert result er usannsynlig gitt at sannsynligheten var $1/6$)

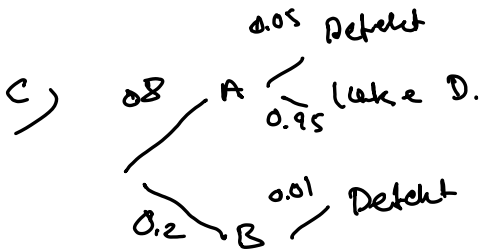
4.

$$\begin{aligned}
 a) \quad & P(120 < X < 130) \\
 &= P\left(\frac{120-123}{\sqrt{121}} < Z < \frac{130-123}{\sqrt{121}}\right) \\
 &= P\left(-3/11 < Z < 7/11\right) \\
 &= \Phi(7/11) - \Phi(-3/11) \\
 &\approx 0.738 - 0.393 \approx \underline{\underline{0.346}}
 \end{aligned}$$



$$\begin{aligned}
 E(Y) &= \int_{-1}^1 y \cdot f(y) dy \\
 &= \int_{-1}^0 y \cdot (-y) dy + \int_0^1 y \cdot y dy
 \end{aligned}$$

$$= \left[-\frac{1}{3} y^3 \right]_{-1}^0 + \left[\frac{1}{3} y^3 \right]_0^1 = \frac{1}{3} + \frac{1}{3} = \underline{\underline{2/3}}$$



D = Defekt
 $\bar{D}$ = keine Defekt

$$\begin{aligned}
 P(A | D) &= \frac{P(A \cap D)}{P(D)} = \frac{0.8 \cdot 0.05}{0.8 \cdot 0.05 + 0.2 \cdot 0.01} \\
 &= \frac{0.04}{0.04 + 0.02} = \frac{0.04}{0.06} = \frac{40}{60} = \frac{20}{30} \approx \underline{\underline{0.95}}
 \end{aligned}$$

5.

$x-y$	$p(x-y)$
0	$6/36$
1	$5/36$
2	$4/36$
$\vdots$	$\vdots$
-1	$5/36$
-2	$4/36$
$\vdots$	$\vdots$

$$a) E(x) = 1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} + \dots + 6 \cdot \frac{1}{6} = \frac{21}{6} = 3.5$$

$$E(x-y) = E(x) - E(y) = \frac{21}{6} - \frac{21}{6} = \underline{\underline{0}}$$

kan også bruke:

$$\begin{aligned} E(x-y) &= 0 \cdot \frac{6}{36} + 1 \cdot \frac{5}{36} + 2 \cdot \frac{4}{36} + 3 \cdot \frac{3}{36} + 4 \cdot \frac{2}{36} + 5 \cdot \frac{1}{36} \\ &\quad + (-1) \cdot \frac{5}{36} + (-2) \cdot \frac{4}{36} + (-3) \cdot \frac{3}{36} + \dots \\ &= 0 \cdot \frac{6}{36} = \underline{\underline{0}} \end{aligned}$$

$$\begin{aligned} b) \text{Var}(x-y) &= \text{Cor}(x-y, x-y) = && \begin{array}{l} x \text{ og } y \\ \text{uavhengige} \end{array} \\ &= \text{Cor}(x, x) - 2 \text{Cor}(x, y) + \text{Cor}(y, y) \\ &= \text{Var}(x) - 2 \text{Cor}(x, y) + \text{Var}(y) && \swarrow \\ &= \text{Var}(x) + \text{Var}(y) && \swarrow \text{symmetri} \\ &= 2 \cdot \left(1^2 \cdot \frac{1}{6} + 2^2 \cdot \frac{1}{6} + \dots + 6^2 \cdot \frac{1}{6} - \left(\frac{21}{6} \right)^2 \right) \end{aligned}$$

$$= 2 \cdot \left(\frac{91}{6} - \left(\frac{21}{6} \right)^2 \right)$$

$$= 2 \left(\frac{91}{6} - \left(\frac{7}{2} \right)^2 \right) = 2 \left(\frac{91}{6} - \frac{49}{4} \right)$$

$$= \frac{91}{3} - \frac{49}{2} = \frac{2 \cdot 91 - 3 \cdot 49}{6} = \frac{35}{6} \approx 5.83$$

$$\begin{aligned} \text{cov}(X+Y, X-Y) &= \text{cov}(X, X) - \text{cov}(X, Y) \\ &+ \text{cov}(Y, X) - \text{cov}(Y, Y) = \text{var}(X) - \text{var}(Y) = 0 \end{aligned}$$

$$c) \quad p(X+Y=2) = \frac{1}{36}$$

(1,1) eneste
mulige utfall

För
eksaml: $p(X+Y=2 | X-Y=1) = 0$

Hvis $X-Y=1$
kan ikke

||

$$X+Y=2$$

$X+Y$ og $X-Y$ ikke uavhengige

(Selv om $\text{cov}(X+Y, X-Y) = 0$).

6. $x = \text{totalenkostning}$ hovedindtækt
 $y = \text{---} \text{---}$ Bekkefrost

a)

$$r = \frac{S_{xy}}{S_x \cdot S_y} = \frac{\sum_{i=1}^{12} (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{12} (x_i - \bar{x})^2} \sqrt{\sum_{i=1}^{12} (y_i - \bar{y})^2}}$$

$$\approx -0.386$$

kalk.

Regressionline (estimer):

$$\hat{\beta} = r \cdot \frac{S_y}{S_x} = -0.386 \cdot \frac{0.099}{0.057} \approx -0.669$$

$$\hat{\alpha} = \bar{y} - \hat{\beta} \cdot \bar{x} = 0.143 - (-0.669) \cdot 0.028$$

$$\approx 0.162$$

||

$$\underline{\underline{y = 0.162 - 0.669x}}$$

b) 90% konfidensintervall for β : $\alpha = 0.10$

$$\hat{\beta} \pm t_{\alpha/2}^{n-2} \cdot SE(\hat{\beta})$$

$$\hat{\beta} = -0.669$$

$$t_{\alpha/2}^{n-2} = t_{0.05}^{10}$$

$$\text{kalk.} \approx 1.812$$

$$SE(\hat{\beta})^2 = \frac{1}{n-2} \frac{\sigma^2}{\sum x_i^2 - n\bar{x}^2}$$

$$\approx \frac{1}{n-2} \frac{S^2}{(n-1)S_x^2}$$

$$= \frac{1}{n-2} \cdot \frac{(n-1)s_x^2(1-r^2)}{(n-1)S_x^2} = \frac{1}{n-2} \frac{s_y^2}{S_x^2} (1-r^2)$$

$$= \frac{1}{10} \frac{0.099^2}{0.057^2} (1 - (-0.386)^2)$$

$$\approx 0.256 \Rightarrow SE(\hat{\beta}) \approx \sqrt{0.256} \approx 0.505$$

Konfidensintervall:

$$\hat{\beta} \pm t_{\alpha/2}^{n-2} \cdot SE(\hat{\beta}) \approx -0.699 \pm 1.812 \cdot 0.505$$

$$= -0.699 \pm 0.916$$

$$\underline{\underline{[-1.615, 0.217]}}$$

c) $H_0: \beta = 0$ ingen sammenheng $\alpha = 0.05$
 $H_1: \beta \neq 0$ sammenheng

Forkastingsområde: $|T| > t_{\alpha/2}^{n-2} = t_{0.025}^{10} = 2.228$

$$T = \frac{\hat{\beta}}{SE(\hat{\beta})} = \frac{-0.699}{0.505} = -1.384 \leftarrow \begin{array}{l} \text{ikke i} \\ \text{forkast.} \\ \text{område} \end{array}$$

Vi beholder H_0 : ingen sammenheng

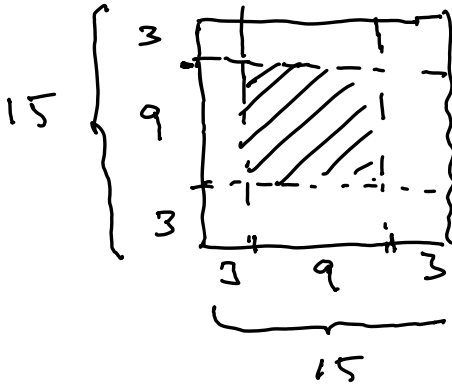
7.



①c

11

Mynten harer innerfor en flis
 = myntens sentrum harer innerfor
 skravert område



Sannsynlighet:

$$p = \frac{9 \cdot 9}{15 \cdot 15} \leftarrow \text{skravert}$$

$$= \left(\frac{9}{15}\right)^2$$

$$= \left(\frac{3}{5}\right)^2 = \frac{9}{25}$$

$$= \underline{\underline{0.36}} \approx 36\%$$