Løsning: Oppgavesett 2

1. Hendelser og sannsynligheter

BK : hun har brystkreft

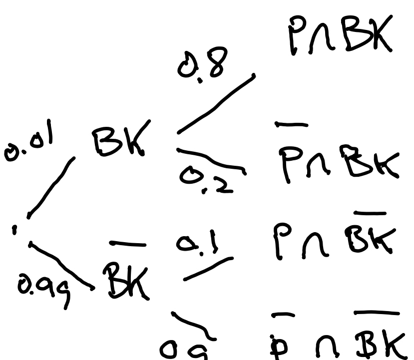
$\overline{BK}$: — " — ikke brystkreft

$p(BK) = 0.01$
$p(\overline{BK}) = 0.99$

P : positiv diagnose (diagnosen viser kreft)

$\overline{P}$: ikke positiv diagnose

$p(P|BK) = 0.80$ $\quad p(\overline{P}|BK) = 0.20$
$p(P|\overline{BK}) = 0.10$ $\quad p(\overline{P}|\overline{BK}) = 0.90$



$$p(P) = p(P \cap BK) + p(P \cap \overline{BK})$$

Sannsynlighet for brystkreft gitt positiv diagnose:

$$p(BK|P) = \frac{p(BK \cap P)}{p(P)}$$

$$= \frac{0.01 \cdot 0.8}{0.01 \cdot 0.8 + 0.99 \cdot 0.1}$$

$$= \frac{0.008}{0.008 + 0.99} = \frac{8}{107} \simeq 0.075 = \underline{\underline{7.5\%}}$$

Sannsynligheten er langt under 50 %.

2. $S = \{KKK, KKM, KMK, MKK, KMM, MKM, MMK, MMM\}$

$2^3 = 8$ utfall, uniform sannsynlighet

$\Downarrow$

hvert utfall har sannsynlighet $1/8$

$$p(\text{mist 2 Kron} \mid \text{mist 1 K}) = \frac{p(\text{mist 2 Kron})}{p(\text{mist 1 kron})} = \frac{4/8}{7/8} = \underline{\underline{4/7}}$$

Merk: $A = \{\text{mist 2 K}\} = \{KKK, KKM, KMK, MKK\}$

$B = \{\text{mist 1 K}\} = \{KKK, KKM, KMK, MKK, KMM, MKM, MMK\}$

$\Downarrow$

$A \cap B = A$

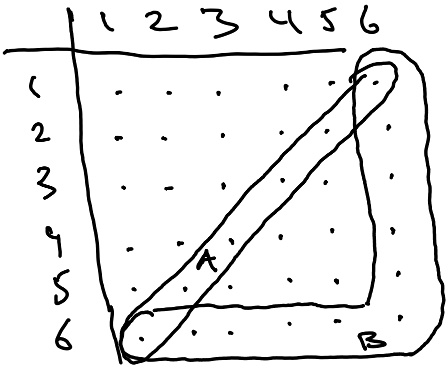
siden

$A \subseteq B$ (alle utfall i A er i B)

$$p(\text{mist 2 K} \mid \text{mist 1 M}) = \frac{p(\text{mist 2 K, minst 1 M})}{p(\text{minst 1 M})}$$

$$= \frac{p(\{KKM, KMK, MKK\})}{p(\text{minst 1 M})} = \frac{3/8}{7/8} = \underline{\underline{3/7}}$$

3.



Det er 36 utfall, alle med sannsynlighet $1/36$

A: summen er 7
B: minst en er 6

a) $p(A) = 6/36 = \underline{1/6}$

b) $p(B) = \underline{11/36}$

c) $p(A \cap B) = \underline{2/36}$

d) $p(A|B) = \frac{p(A \cap B)}{p(B)} = \frac{2/36}{11/36} = \underline{\underline{2/11}}$

e) $p(B|A) = \frac{p(A \cap B)}{p(A)} = \frac{2/36}{6/36} = 2/6 = \underline{\underline{1/3}}$

f) $p(A^c|B) = 1 - p(A|B) = 1 - 2/11 = \underline{\underline{9/11}}$

g) $p(B^c|A) = 1 - p(B|A) = 1 - 1/3 = \underline{\underline{2/3}}$

h) $p(A^c \cap B^c) = \frac{15+6}{36} = \frac{21}{36} = \underline{\underline{\frac{7}{12}}}$

Det er 15+6 utfall i $A^c \cap B^c$, dvs utenfor både A og B.

4. A = summen er 7

B = minst en er 6

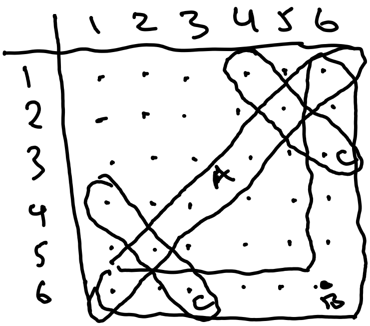
C = differanse mellom høyeste og laveste er 3

a) $p(A \cap B) = 2/36 = 12/216$ ← se oppg. 3

$p(A) \cdot p(B) = 1/6 \cdot 11/36 = 11/216$

A og B er <u>ikke</u> uavhengige $\quad \boxed{p(A \cap B) \neq p(A) \cdot p(B)}$

c)



$$P(A|C) = \frac{p(A \cap C)}{p(C)} = \frac{2/36}{6/36} = 2/6 = \underline{1/3}$$

$$P(A) = 6/36 = \underline{1/6}$$

$\boxed{p(A|C) \neq p(A)}$ A og C er <u>ikke</u> uavhengige

b) $$p(B|C) = \frac{p(B \cap C)}{p(C)} = \frac{2/36}{6/36} = 2/6 = \underline{1/3}$$

$p(B) = \underline{11/36}$

B og C er <u>ikke</u> uavhengige $\quad \boxed{P(B|C) \neq p(B)}$

Merk: Kan bruke $P(A \cap B) = P(A) \cdot P(B) \Longleftrightarrow P(A|B) = P(A)$

5. $(x+y)^7 = \underbrace{(x+y)(x+y)\cdots(x+y)}_{7 \text{ faktorer}}$

$$= x^7 + c_1 \cdot x^6 y + c_2 \cdot x^5 y^2 + \dots + y^7$$

Hver ledd inneholder $x^a y^b$ ved $a+b=7$ siden det er 7 faktorer som skal multipliseres ut.

Koeffisienten c_i foran $x^a y^b$ er hvor mange måter vi kan velge ut a kopier av x fra de 7 faktorene, altså $\binom{7}{a}$. Dette er også lik $\binom{7}{b}$, siden det er antall måter å velge ut b kopier av y.

Koeffisienten foran x^3y^4 er derfor

$$\binom{7}{3} = \binom{7}{4} = \frac{\cancel{7\cdot6\cdot5}}{\cancel{3\cdot2\cdot1}} = \underline{\underline{35}}$$

Binomialformel:

$$(x+y)^7 = x^7 + \binom{7}{1}x^6y + \binom{7}{2}x^5y^2 + \binom{7}{3}x^4y^3 + \binom{7}{4}x^3y^4 + \binom{7}{5}x^2y^5 + \binom{7}{6}xy^6 + y^7$$

$$= x^7 + 7x^6y + 21x^5y^2 + 35x^4y^3 + 35x^3y^4 + 21x^2y^5 + 7xy^6 + y^7$$

b. $\binom{n}{0} = 1$ (per defn.)

$$\binom{n}{1} = \frac{n}{1} = n$$

c) $\binom{n}{r} = \frac{n!}{r!\,(n-r)!}$

$$\binom{n}{n-r} = \frac{n!}{(n-r)!\cdot(n-(n-r))!} = \frac{n!}{(n-r)!\,r!}$$

d) $\binom{n-1}{r} + \binom{n-1}{r-1} = \frac{(n-1)!}{r!\,(n-1-r)!} + \frac{(n-1)!}{(r-1)!\,(n-r)!}$

$$= \frac{(n-1)!\cdot(n-r)}{r!\,(n-r-1)!\cdot(n-r)} + \frac{(n-1)!\cdot r}{(r-1)!\,(n-r)!\cdot r}$$

$$= \frac{(n-1)!\cdot\left[n-\cancel{r}+\cancel{r}\right]}{r!\,(n-r)!} = \frac{n!}{r!\,(n-r)!}$$

Legger sammen brøken ved å utvide til fellesnevner

7. Antar at A, B uavhengige, dvs

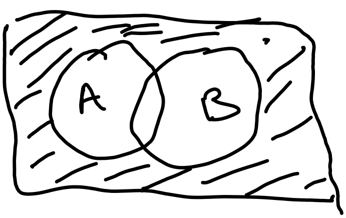
$$\underline{P(A \cap B) = P(A) \cdot P(B)}$$

$$p(A^c \cap B^c) = p((A \cup B)^c) = 1 - p(A \cup B)$$

$$= 1 - (p(A) + p(B) - p(A \cap B))$$

$$= 1 - p(A) - p(B) + p(A \cap B)$$

$$\underline{= 1 - p(A) - p(B) + p(A) \cdot p(B)}$$



Skravert $= A^c \cap B^c$

$= (A \cup B)^c$

(uttrykt ved $p(A), p(B)$)

$$p(A^c \cap B^c) = 1 - p(A) - p(B) + p(A) \cdot p(B)$$

$$p(A^c) \cdot p(B^c) = (1 - p(A))(1 - p(B))$$

$$= 1 - p(A) - p(B) + p(A) \cdot p(B)$$

$p(A^c \cap B^c) = p(A^c) \cdot p(B^c) \Rightarrow$ A^c, B^c er uavhengige

8. (Vanskelig)

n=2: $p(A_1 \cup A_2) = 1 - p(\overline{A_1 \cup A_2})$

$= 1 - p(\overline{A_1} \cap \overline{A_2})$

$= 1 - p(\overline{A_1}) \cdot p(\overline{A_2})$

fra Opps. 7

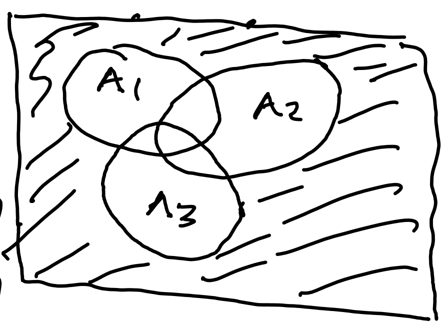
n=3: $p(A_1 \cup A_2 \cup A_3)$

$= 1 - p(\overline{A_1 \cup A_2 \cup A_3})$

$= 1 - p(\overline{A_1 \cup A_2} \cap \overline{A_3})$

$= 1 - p(\overline{A_1 \cup A_2}) \cdot p(\overline{A_3})$

$= 1 - p(\overline{A_1}) \cdot p(\overline{A_2}) \cdot p(\overline{A_3})$



$\overline{A_1 \cup A_2 \cup A_3} = \overline{(A_1 \cup A_2)} \cap \overline{A_3}$

$(A_1 \cup A_2)$ og A_3 er uavhengige, siden

$p((A_1 \cup A_2) \cap A_3)$

$= p(A_1 \cap A_3) + p(A_2 \cap A_3)$

$= p(A_1) \cdot p(A_3) + p(A_2) \cdot p(A_3)$

$= (p(A_1) + p(A_2)) \cdot p(A_3)$

$= p(A_1 \cup A_2) \cdot p(A_3)$

Merk: $A_1 \cup A_2$ og A_3 uavhengige

$\Downarrow$

Kan bruke $n=2$ tilfellet

9. Bruker resultatet fra Oppg 8:

A_1 : smittet av person 1
A_2 : — " — 2
$\vdots$
A_n : — " — — n

$$\left.\begin{matrix} A_1 \\ A_2 \\ \vdots \\ A_n \end{matrix}\right\} \quad p(A_i) = \frac{1}{200} \Leftrightarrow p(\overline{A_i}) = \frac{199}{200}$$

$A = A_1 \cup A_2 \cup \cdots \cup A_n$

Smittet av minst én av person 1, 2, ..., n.

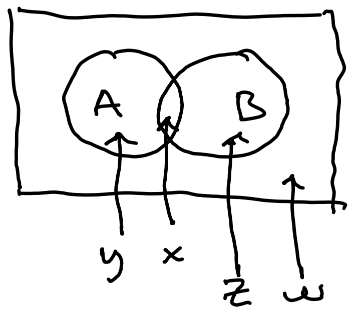
a) $\underline{n=10}$: $p(A) = p(A_1 \cup \cdots \cup A_{10})$
$$= 1 - p(\overline{A_1}) \cdot p(\overline{A_2}) \cdot \ldots \cdot p(\overline{A_{10}})$$
$$= 1 - \frac{199}{200} \cdot \frac{199}{200} \cdot \ldots \cdot \frac{199}{200}$$
$$= 1 - (199/200)^{10} \simeq \underline{0.049}$$

b) $\underline{n=100}$: $p(A) = 1 - (199/200)^{100} \simeq \underline{0.394}$

c) $\underline{n=200}$: $p(A) = 1 - (199/200)^{200} \simeq \underline{0.633}$

d) $p(A) = \underline{1 - (199/200)^n}$

10.



$$\left.\begin{aligned} p(A\cap B) &= x \\ p(A\cap \bar{B}) &= y \\ p(\bar{A}\cap B) &= z \\ p(\bar{A}\cap \bar{B}) &= w \end{aligned}\right\}$$ kaller de ulike sannsynligh. x, y, z, w

$\boxed{x+y+z+w=1}$

<u>Venstre side:</u>

$p(A\cap B) = x$

<u>Høyre side:</u>

$$\begin{aligned} &p(A)+p(B)-1 \\ &= (x+y)+(x+z)-1 \\ &= 2x+y+z-(x+y+z+w) \\ &= x-w \end{aligned}$$

$\boxed{w \geq 0}$ $\longrightarrow$ Siden $x \geq x-w$,

får vi:

$$\boxed{p(A\cap B) \geq p(A)+p(B)-1}$$