Løsning: Oppgavesett 7

1.

a) X = antall kron på n kast

$\bar{X}$ = andelen kast med kron

b) $E(x_i) = 1 \cdot \frac{1}{2} + 0 \cdot \frac{1}{2} = \underline{\underline{1/2}}$

$Var(x_i) = E(x_i^2) - E(x_i)^2$

$= \left(1^2 \cdot \frac{1}{2} + 0^2 \cdot \frac{1}{2}\right) - (1/2)^2 = 1/2 - 1/4 = \underline{\underline{1/4}}$

Kunne også ha brukt at $X_i \sim Binom(1, 1/2)$
med $n = 1$, $p = 1/2$.

c) $\mu_X = E(X) = 100 \cdot 1/2 = \underline{\underline{50}}$

$\sigma_X^2 = Var(X) = 100 \cdot 1/4 = 25 \Rightarrow \sigma_X = \sqrt{25} = \underline{\underline{5}}$

Kunne også ha brukt at $X \sim Binom(n, p)$
med $n = 100$, $p = 1/2$.

d) $\mu_{\bar{X}} = E(\bar{X}) = \frac{1}{n} \cdot E(X) = \frac{50}{100} = \underline{\underline{1/2}}$

$\sigma_{\bar{X}}^2 = Var(\bar{X}) = \left(\frac{1}{n}\right)^2 \cdot Var(X) = \frac{5^2}{100^2}$

$\Rightarrow \sigma_{\bar{X}} = \frac{5}{100} = \underline{\underline{0.05}}$

e) $|\bar{X} - 0.50| > 0.1 \Leftrightarrow \bar{X} > 0.6$ eller $\bar{X} < 0.4$

$\bar{X} > 0.5 + 0.1$ $\quad \bar{x} < 0.4 - 0.1$

$$Z = \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}}$$

(n = 100)

$\Updownarrow$

$Z > 2$ eller $Z \le -2$

siden $0.1 = 2 \cdot 0.05$
$= 2 \cdot \sigma_{\bar{X}}$

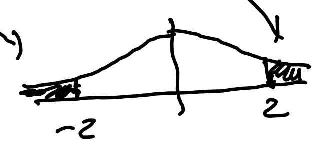
Chebychevs ulikhet:

$p(|Z| > 2) \le 1/2^2 = 1/4 = \underline{\underline{0.25}}$

f) $\bar{X}$ normalfordelt:

$p(|Z| > 2) = p(Z < -2) + p(Z > 2)$

$= 2 \cdot G(-2) \simeq \underline{\underline{0.046}}$



g) Vi har antatt at myntkastene er uavhengige. Det virker rimelig.

2. a) b) : Se Oppgave I

c) $\mu_X = 10.000 \cdot \frac{1}{2} = \underline{\underline{5000}}$

$\sigma_X^2 = 10.000 \cdot \frac{1}{4} = 2500 \Rightarrow \sigma_X = \sqrt{2500} = \underline{\underline{50}}$

d) $\mu_{\bar{X}} = \underline{\underline{1/2}} \quad \sigma_{\bar{X}}^2 = \frac{1/4}{10.000} \Rightarrow \sigma_{\bar{X}} = \frac{1}{200} = \underline{\underline{0{,}005}}$

e) $0.1 = 20\,\sigma_{\bar{X}} \Rightarrow |Z| \geq 20$

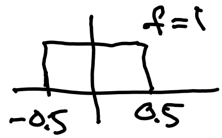
Chebyshev: $P(|Z| \geq 20) \leq \frac{1}{20^2} = \frac{1}{400} = \underline{\underline{0{,}0025}}$

f) $P(|Z| \geq 20) = P(Z < -20) + P(Z > 20)$

$= 2 \cdot G(-20) \simeq 5.5 \cdot 10^{-8} \simeq \underline{\underline{0}}$

g) : Se oppg. I

3. a) R_1 : Uniform $[-0.5, 0.5]$



$E(R_i) = \int_{-0.5}^{0.5} r \, dr = \left[\frac{1}{2} r^2\right]_{-0.5}^{0.5} = \underline{\underline{0}}$

$E(R_i^2) = \int_{-0.5}^{0.5} r^2 \, dr = \left[\frac{1}{3} r^3\right]_{-0.5}^{0.5} = \frac{1}{3}\left(\frac{1}{8} - \left(-\frac{1}{8}\right)\right)$

$= \frac{1}{3} \cdot \frac{2}{8} = \frac{1}{12}.$

$\Rightarrow \mathrm{Var}(R_i) = \frac{1}{12} - 0 = \frac{1}{12} \Rightarrow \sigma_{R_1} = \sqrt{\frac{1}{12}} \simeq \underline{\underline{0.29}}$

b) $R = R_1 + R_2$

$E(R) = 2 \cdot E(R_1) = 2 \cdot 0 = \underline{\underline{0}}$

$Var(R) = 2 \cdot Var(R_1) = 2 \cdot \frac{1}{12} = \frac{1}{6} \Rightarrow \sigma_R = \sqrt{\frac{1}{6}}$

$\simeq \underline{\underline{0.41}}$

c) $E(R) = 5 \cdot 0 = \underline{\underline{0}}$

$Var(R) = 5 \cdot \frac{1}{12} = 5/12 \Rightarrow \sigma_R = \sqrt{5/12} \simeq \underline{\underline{0.65}}$

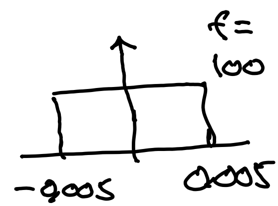
d) $E(R) = 10 \cdot 0 = \underline{\underline{0}}$

$Var(R) = 10 \cdot 1/12 = 5/6 \Rightarrow \sigma_R = \sqrt{5/6} \simeq \underline{\underline{0.91}}$

e) $E(R) = 100 \cdot 0 = \underline{\underline{0}}$

$Var(R) = 100 \cdot 1/12 = \frac{100}{12} = \frac{50}{6} = \frac{25}{3} \Rightarrow \sigma_R = \sqrt{25/3} \simeq \underline{\underline{2.89}}$

<u>4.</u> $E(R_1) = \int_{-0{,}005}^{0{,}005} r \cdot 100\,dr$



$= \frac{1}{2} r^2 \cdot 100 \Big|_{0{,}005}^{0{,}005} = \underline{\underline{0}}$

$Var(R_1) = E(R_1^2) - 0^2 = \int_{-0{,}005}^{0{,}005} r^2 \cdot 100\,dr$

$= \frac{100}{3} r^3 \Big|_{-0{,}005}^{0.005} = \frac{100}{3} \cdot \left(\left(\frac{1}{200}\right)^3 - \left(-\frac{1}{200}\right)^3 \right)$

$= \frac{100}{3} \cdot \frac{2}{(200)^3} = \frac{1}{120000} \Rightarrow \sigma_{R_1} = \sqrt{1/120{,}000}$

$R = R_1 + R_2 + \ldots + R_{100}$:

$E(R) = 100 \cdot 0 = \underline{\underline{0}}$

$Var(R) = 100 \cdot 1/120000 = 1/1200 \Rightarrow \sigma_R = \sqrt{1/1200}$

$\simeq 0.029$

R tilnærmet normalfordelt $\underline{N(0, 1/15)}$

a) $p(|R| > 0.05) = p(R < -0.05) + p(R > 0.05)$

$= p\left(Z < \frac{-0.05}{\sigma_R}\right) \cdot 2 \simeq 2 \cdot G(-1.73)$

$\simeq \underline{\underline{0.083}}$

b) $p(|R| > 0.01) = 2 \cdot G\left(\frac{-0.01}{\sigma_R}\right) \simeq \underline{\underline{0.729}}$

c) $p(|R| > 0.005) = 2 \cdot G\left(\frac{-0.005}{\sigma_R}\right) \simeq \underline{\underline{0.862}}$

d) $p(|R| > 0.001) = 2 G\left(\frac{-0.001}{\sigma_R}\right) \simeq \underline{\underline{0.973}}$

e) $|R| > 0.005 \Leftrightarrow$ Stemmer ikke overens med to desimaler

$p \simeq \underline{\underline{0.862}} = \underline{\underline{86.2\%}}$

5.

a) $X \sim \text{Binom}(n = 360,\ p = 1/6)$

b) $E(X) = np = 360 \cdot 1/6 = \underline{\underline{60}}$

$Var(X) = npq = 360 \cdot 1/6 \cdot 5/6 = 50$

$\Rightarrow \sigma_X = \sqrt{50} = \underline{\underline{5\sqrt{2}}}$

c) <u>X tilnærmet normalfordelt:</u>

$p(X \leq 55) = p\left(Z \leq \frac{55{,}5 - 60}{5\sqrt{2}}\right)$

$\simeq G(-0{,}636) \simeq \underline{\underline{0.262}} = 26{,}2\,\%$

d) $p(X \geq 70) = 1 - p(X \leq 69)$

$\simeq 1 - p\left(\frac{69.5 - 60}{\sqrt{50}}\right) \overset{\sim}{=} 1 - G(1{,}34)$

$\simeq \underline{\underline{0.090}} = \underline{\underline{9\,\%}}$

6. a) $E(X+Y) = 2 + 5 = \underline{\underline{7}}$

$Var(X+Y) = 9 + 16 + 2 \cdot 5 = 35$

$\left.\right\}$ $X+Y \sim N(\underset{\mu}{7}, \underset{\sigma^2}{35})$

$E(2X+3Y) = 2 \cdot 2 + 3 \cdot 5 = \underline{19}$

$Var(2X+3Y) = 4 \cdot 9 + 9 \cdot 16 + 2 \cdot 2 \cdot 3 \cdot 5$

$= 36 + 144 + 60 = 240$

$\left.\right\}$ $2X+3Y \sim N(\underset{\mu}{19}, \underset{\sigma^2}{240})$

b) $p(X+Y \leq 5) = p\left(Z \leq \frac{5-7}{\sqrt{35}}\right)$

$= G(-0.34) \simeq \underline{\underline{0.368}}$

c) $p(0 < 2X+3Y < 2) = p\left(\frac{0-19}{\sqrt{240}} \leq Z \leq \frac{2-19}{\sqrt{240}}\right)$

$\simeq G(0.77) - G(-1.226)$

$\simeq 0.1362 - 0.1100 \simeq \underline{\underline{0.026}}$

7.

X_i : kast i

$X = X_1 + X_2 + \ldots + X_{79}$

$E(X_i) = 1 \cdot \frac{1}{6} + \ldots + 6 \cdot \frac{1}{6} = \frac{7}{2} = \underline{3.5}$

$Var(X_i) = 1^2 \cdot \frac{1}{6} + \ldots + 6^2 \cdot \frac{1}{6} - (7/2)^2 = \frac{91}{6} - \frac{49}{4}$

$= \frac{35}{12} \Rightarrow \sigma_X = \sqrt{35/12}$

$E(X) = 79 \cdot 3.5 = \underline{\underline{276.5}}$

$Var(X) = 79 \cdot 35/12 \Rightarrow \sigma_X \simeq \underline{15.18}$

$p(X \leq 300) = p\left(Z \leq \frac{300+0.5-276.5}{\sigma_X}\right)$

$\simeq G(1.58) \simeq \underline{\underline{0.943}} = \underline{\underline{94.3\%}}$